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GENERAL ELECTRIC COMPANY

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Title: Calculation of Receiving Tube Constants

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Calculation of Receiving Tube Constants

1. Introduction

This report covers the work done in a study of receiving tube characteristics begun in June 1946, and could serve as a starting point for future investigations.

2. Diodes

No investigation was made of diode characteristics. For a discussion of diodes see Terman (1)*.

3. Triodes

(a) Amplification Factor

A number of charts were prepared which simplify the transition from known tubes to new designs, and which also may be used to calculate the amplification factor of triodes quickly.

The charts are based upon the Salzberg (2) formula for the amplification factor. Use of these charts will probably demonstrate a need for their extension or a change in their form in order to simplify and enlarge the scope of their application. The charts and instructions for their use may be found in Data Folder No. 86914. (5)

Due to certain assumptions made in its derivation, the Salzberg formula becomes inaccurate when the grid-cathode spacing is less than the grid wire spacing. This case was studied by Fremlin (6)

Numbers in parentheses refer to items in the bibliography and a formula for the "electrostatic Durchgriff" was derived by him for plane structures. This formula is so complex that it is almost unuseable, and will not be given here. It does, however, demonstrate that the amplification factor varies along the cathode, rising to maximum values when the portion considered is directly under a grid wire.

(b) Plate Current and Mutual Conductance

The plate current and mutual conductance of triodes have been calculated by the use of an equivalent diode. The plate of this diode is placed in the position of the triode grid, and the plate voltage is given as $(u\ E_g + E_p)/(1 + u)$. The Child-Langmuir Law then gives the plate current, and, by differentiation, the mutual conductance.

Fremlin (6) has demonstrated that this method is subject to error, and that the plate current in plane structures is given by

$$i_p = \frac{2.34 \times 10^{-6}\left[E_g + \frac{1}{\mu}E_p\right]^{3/2}}{\left[l_g^{4/3} + \frac{1}{\mu} l_p^{4/3}\right]^{3/2}} \ \frac{\text{Amps.}}{\text{Unit Area}} \quad \text{(I)}$$

in which

l_g = grid-cathode spacing

l_p = plate-cathode spacing

and for circular structures

$$i_p = \frac{14.7 \times 10^{-6}\left[E_g + \frac{1}{\mu}E_p\right]^{3/2}}{\left[(r_g \beta_{cg}^2)^{2/3} + \frac{1}{\mu}(r_p \beta_{cp}^2)^{2/3}\right]^{3/2}} \ \frac{\text{Amps}}{\text{Unit Length}} \quad \text{(II)}$$

*The "Durchgriff", from the German is literally translated as the "reaching through"; i.e., the plate reaches through the grid. In a triode, the "Durchgriff" is the reciprocal of the amplification factor.

where

rg = grid radius

rp = plate radius

β_{cg}^2 = β^2 from Langmuir's emission equation based on cathode and grid radii.

β_{cp}^2 = β^2 based on cathode and plate radii.

Differentiation of eqs. (I) and (II) gives as the mutual conductance of the plane structure

$$g_m = \frac{3.50\times10^{-6}\left[E_g+\frac{1}{\mu}E_p\right]^{1/2}}{\left[l_g^{4/3}+\frac{1}{\mu}l_p^{4/3}\right]^{3/2}}\ \frac{\text{mhos}}{\text{Unit Area}} \qquad \text{(III)}$$

and, for the circular structure

$$g_m = \frac{22.0\times10^{-6}\left[E_g+\frac{1}{\mu}E_p\right]^{1/2}}{\left[(r_g\beta_{cg}^2)^{2/3}+\frac{1}{\mu}(r_p\beta_{cp}^2)^{2/3}\right]^{3/2}}\ \frac{\text{mhos}}{\text{Unit Length}} \qquad \text{(IV)}$$

During his investigation Fremlin built an experimental plane structure triode in which the grid-cathode and plate-cathode spacing could be varied at will. With this tube he checked the equations for plane structures. Good agreement was obtained between measured and calculated values of the characteristics, even in the case of small grid-cathode spacings.

The writer attempted to correlate the results of eqs. (I) and (III) with data taken by Russ on an experimental plane triode. The electrodes of this tube were discs of approximately equal diameter. The calculated values of ip and gm, which were made using measured values of u and grid contact potential, differed from the measured values by ratios as great as three to one.

In evaluating these results one must consider the possible influence of edge effects. The Fremlin tube incorporated means for eliminating this variable, while the Russ tube did not. In the Russ tube the length of the edge was much larger than that of conventional tubes which suggests that edge effects may play an important part in determining the characteristics of the tube. In any event, it should prove instructive to obtain experimental results with a tube similar to that of Russ, but with means incorporated to eliminate edge effects. Such an investigation would help to determine the magnitude of the errors due to this variable.

A comparison between calculated and published values was then made for a number of cylindrical element tubes. The dimensions of the tubes were taken from the factory drawings. The difference between the calculated and published values were as large as those found in the plane structure tube. It was thought that in this case the errors were due principally to four factors; first, deviation of the structures from circular form; second, the presence of the grid side rods, with attendent suppression of the flow of plate current through adjacent sections of the grid; third, deviation of the actual grid contact potential from the values assumed; and fourth, deviation of the average characteristics from the published values.

The effect of the grid side rods was then taken into account by the method of Yeh (7), but large differences were still present.

A semi-empirical approach was then decided upon. Due to the presence of the grid side rods, it was decided to regard the tube as a plane triode, with a central cathode, and parallel grid and plate structures on either side of the cathode. The cathode emission area was taken, as a first approximation, to be equal to the projected area of the cathode coating for each side of the triode. The formulae for the plane structure tube, eqs. (I) and (III) were then used. The results were much better, especially in the case of tubes with low amplification factors.

For comparative purposes, the characteristics were also calculated with formulae used by L. U. Hemvas (8), which are

$$i_p = \frac{14.7 \times 10^{-6}\left[E_g + \frac{1}{\mu} E_p\right]^{3/2}}{\left[r_g - r_c\right]\left[1 + \frac{1}{\mu}\right]^{3/2}} \ \frac{\text{Amps}}{\text{Unit Length}} \qquad \text{(V)}$$

and

$$g_m = \frac{22.0 \times 10^{-6}\left[E_g + \frac{1}{\mu} E_p\right]^{1/2}}{\left[r_g - r_c\right]\left[1 + \frac{1}{\mu}\right]^{3/2}} \ \frac{\text{mhos}}{\text{Unit Length}} \qquad \text{(VI)}$$

The results of calculations made by the two methods are tabulated below. In this tabulation a negative sign indicates that the calculated value is less than the published value. The percentage error given for each tube is the average of the percentage errors for several different operating points.

Comparison of Hamvas and Fremlin Formulae

Type	Hamvas % Error		Fremlin % Error		
	ip	gm	ip	gm	
6AQ6	118.5	66.5	121	57.5	
12AT6	58.5	13.7	132	66.8	High u
6SF5	59	-11.3	41.6	-20.9	Tubes
6SQ7	43	5	52	11	
6J5	7.3	0	9.4	0.5	
6C4	41	53	9.5	8	Low u
9002	-40.6	-27.2	-21.5	-18	Tubes

The larger errors encountered in the high u tubes may be due in part to deviation of the grid contact potential from the value assumed in the calculations; i.e., one volt. It will be found, however, that if the assumed contact potential is corrected to a value which gives the correct plate current, the mutual conductance calculation will remain in error. It would seem, therefore, that sources of error other than deviation in the contact potential exist.

The reasons for these errors may be listed in part as:

1. Deviation of the tube geometry from that specified.
2. Maxwellian distribution of the emitted electrons, not only in the magnitude of velocity, but also in the angle between the path and the cathode surface.
   (Note: The Child-Langmuir formulae are based on the assumption that electrons are emitted from the cathode with essentially zero velocity.)

3. Deviation of the exponent from the value of three halves used in the Child-Langmuir Law.*

Another error in the equivalent diode type of formula may be shown as follows:

Let us divide eq. (III) by eq. (I), this gives us an expression for the gm/ip ratio.

$$\frac{g_m}{i_p} = \frac{2}{3\left[E_g + \frac{1}{\mu} E_p\right]} \qquad \text{(VII)}$$

Eq. (VII) implies that the gm/ip ratio is independent of the tube structure if the amplification factor is maintained constant. This is contrary to experience reported by Hamvas that the gm/ip ratio is increased when the size of the grid wire is decreased and the grid wire spacing is decreased the necessary amount to maintain the u constant.

On this basis we may conclude that the use of the equivalent diode with its plate voltage determined only by the amplification factor and electrode voltages of the triode is not completely correct, and that the dimensions of the grid structure should in some manner be included.

Multigrid Tubes

Only a beginning was made on the problem of multigrid tubes. The method of Hamvas (8) may be used to evaluate the equivalent diode

* The finite velocity of the emitted electrons causes the exponent of the voltage in the plate current formula to deviate from the value of three halves given by Child and Langmuir. This is most noticeable at small plate currents. In fact, it will be found that the relationship changes from a power function to an exponential function at very low plate currents.

voltage and the space current, although the possibility of revision to include the methods of Fremlin (5) exists.

The ratio fo screen grid current to plate current must be known in addition to the space current if the characteristics of the tube are to be determined. Hamvas has derived an expression for this ratio from a number of quite reasonable assumptions. He gives for the tetrode, neglecting secondary emission

$$\frac{i_{sg}}{i_p} = C_1 \left[\frac{\mu_2 E_{sg} + E_p}{(1+\mu_2) E_p} \right]^{3/2} \qquad \text{(VIII)}$$

Where C_1 is a function of the tube geometry

u_2 is the amplification factor of the screen grid-plate triode.

and for the pentode

$$\frac{i_{sg}}{i_p} = C_2 \left[\frac{\mu_2 (1+\mu_3) E_{sg} + E_p}{(1+\mu_2)(1+\mu_3) E_p} \right]^{3/2} \qquad \text{(IX)}$$

Where u_2 is the amplification factor of the screen grid-suppressor grid triode

u_3 is the amplification factor of the suppressor grid-plate triode.

Rothe and Kleen (9) divide the plate characteristic of a multigrid tube into two regions, one in which some of the electrons injected into the screen-plate space return to the screen, and another in which all such electrons are drawn to the plate. In the first region they give the ratio of plate current to cathode current as

$$\frac{i_p}{i_K} = C_3 \sqrt{\frac{E_P}{E_{sg}}} \qquad \text{(X)}$$

and in the second region, for the tetrode, the plate-screen grid current ratio as

$$\frac{i_p}{i_{sg}} = C_4 \sqrt{\frac{E_p}{E_{sg}}} \qquad \text{(XI)}$$

Eq. (XI) is modified in the case of the pentode to

$$\frac{i_p}{i_{sg}} = C_4 \sqrt{\frac{aE_p + dE_{sg}}{(1+a+d)\,E_{sg}}} \qquad \text{(XII)}$$

where

a is the "Durchgriff" of the plate through the suppressor grid

d is the "Durchgriff" of the screen grid through the suppressor grid.

From eq. (XII) Rothe and Kleen derive a semi-empirical relationship for pentodes,

$$R_p = b \cdot \frac{E_p}{i_p} \qquad \text{(XIII)}$$

where b is a constant determined by the geometry of the tube.

From eq. (XIII), since $R_p = \dfrac{\partial E_p}{\partial i_p}$, they get by integration

$$i_p = C_5\, f(E_g, E_{sg}) \sqrt[b]{E_p} \qquad \text{(XIV)}$$

where $f(E_g, E_{sg})$ is a function of the control and screen grid voltages and is independent of plate voltage.

While time was insufficient to verify experimentally any of the above formulae it is evident that much can be done to investigate their validity. As a preliminary step, data taken from published curves was used to plot curves of i_p/i_{sg} vs. $\sqrt{E_p}$, and i_p/i_k vs. $\sqrt{E_p}$. The plotted curves were essentially broken straight lines, which would satisfy eqs. (X), (XI), and (XII). Similarly, plots of ip vs. Ep were made on semilog paper. In order to satisfy eq. (XIV) such families of curves should consist of parallel straight lines. No conclusive results were obtained. If any such curves are to be used as a basis for conclusions regarding the formulae presented, they should be based upon experimental rather than published information.

Beam Tubes

No investigation was made of beam tubes, but a partial survey was made of the available literature.

Terman (10) gives formulae for the potential distribution between the screen grid and plate for various conditions of plate and screen voltage. These formulae, however, do not appear to be applicable to design problems. He also gives an expression for the maximum plate current which may be passed through the screen grid-plate region. This is

$$i_{P\,max} = \frac{2.33 \times 10^{-6} \left[E_{sg}^{1/2} + E_{P}^{1/2}\right]^{3}}{l_{gp}^{2}} \; \frac{\text{Amps.}}{\text{Unit Area}} \qquad \text{(XV)}$$

where lgp is the grid plate distance and the area is measured in the same system of units as lgp.

While the formulae given by Terman are not identified as to the kind of structure to which they apply, the form of the expressions indicate that they are applicable to plane structure tubes.

Schade (11) gives an excellent exposition of the operation and design considerations of beam tetrodes, but with little concrete data.

Converters and Mixers

No work was done on converters or mixer tubes.

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Nomenclature

a - "Durchgriff" of plate through suppressor grid

b - a constant

C_1, C_2, C_4, C_5 - constants

d - "Durchgriff" of screen grid through suppressor grid

Eg - control grid voltage

Ep - plate voltage

Esg - screen grid voltage

ip - plate current

ik - cathode current

isg - screen grid current

gm - mutual conductance

lg - grid-cathode spacing to centerline of grid wire

lp - plate-cathode spacing

lgp - grid-plate spacing

rg - radius of grid to centerline of grid wire

rp - inside radius of plate

rc - radius of cathode over cathode coating

Rp - plate resistance

u - amplification factor

β^2_{cg} - β^2 from Langmuir's emission equation based on cathode and grid radii

β^2_{cp} - β^2 based on cathode and plate radii

Bibliography

Diodes

(1) F. E. Terman

"Radio Engineers Handbook" 1st Ed. McGraw-Hill Book Co., 1943, pp. 286-289.

Gives curves of diode plate current for plane and cylindrical diodes.

Triodes

(2) Bernard Salzberg

"Formulas for the Amplification Factor of Triodes" Proc. I.R.E., Vol. 30, p. 134, March 1942.

A mathematical derivation of the amplification factor of triodes in which the grid wire spacing is comparable with the grid wire diameter.

(3) F. B. Vodges and F. R. Elder

"Formulas for the Amplification Constant for Three Element Tubes" Phys. Rev., Vol. 24, p. 683, December, 1924.

A mathematical derivation of the amplification factor of triodes in which the grid wire spacing is large in comparison with the grid wire diameter.

(4) Y. Kusunose

"Calculation of the Characteristics and the Design of Triodes" Proc. I.R.E., Vol. 17, No. 10, pp. 1706-1749, October, 1929.

Application of the work of Vodges and Elder (3) to actual tube structures with numerous examples.

A summary of references (3) and (4) will be found in Terman (1) pp. 304-307

(5) J. E. Fowler

"Amplification Factor Charts for Triodes", D. F. - 86914, Electronic Tube Engineering Div., September 1946.

(6) J. H. Fremlin

"Calculation of Triode Constants", Electrical Communications, Vol. 18, No. 1, July 1939.

Derivation of plate current and mutual conductance formulae for triodes with experimental confirmation of results.

(7) Chai Yeh

"The Effect of Grid-Support Wires on Focusing Cathode Emission" Proc. I.R.E., Vol. 30, No. 7 pp. 444-447, July 1946.

A mathematical study which gives a formula to determine the angle on the cathode over which emission is prevented by the presence of grid support wires.

Multigrid Tubes

(8) L. U. Hamvas

"Design of Multigrid Tubes". See appendix.

(9) H. Rothe and W. Kleen

"Stromverteilung" (Current Division) Die Telefunken-Rohre, No. 6 pp. 1-23, March 1936. (In German)

A study of the division of space current between the positive grid and the anode, with a study of pentode characteristics.

Beam Tubes

(10) Terman

(1) pp. 289-292

Formulae for the potential distribution between the screen grid and plate in beam tubes, with an expression for the maximum plate current possible for given conditions.

(11) O. H. Schade

"Beam Power Tubes", Proc. I.R.E. Vol. 26, No. 2, pp. 145-169, February 1938.

Description of operation and theory of beam tetrode.

DESIGN OF MULTIGRID TUBES

This paper attempts to give a few formulae into the hands of the vacuum tube designer, without the precisions of purely theoretical computation. The derived formulae hold only within practical limits and under circumstances which are common when working on or with receiving tubes. They will give, under ordinary circumstances, solving such problems which the designer has to face daily, very accurate results.

I. EMISSION CURRENT OF MULTIGRID TUBES

In the well known emission formula (Langmuir)

$$I_s = KE^{3/2}$$

The E is the effective plate voltage which was computed (Barkhausen) in the following way:

There is a homogeneous electrical field in the neighborhood of the cathode, which should be marked F. If we disregard the influence of the space charge, we can compute F for a distance r from the cathode as $F = \frac{2Q}{r}$

where Q is the charge upon the unit length of the cathode. This Q might be expressed in case of cylindrical diodes as

(1) $$Q = CE$$

where C is the capacity and E the potential difference between the two electrode In case of cylindrical triode we can say similarly:

(2) $$Q = C_g E_g + C_p E_p$$

where C_g and C_p are the capacities of grid and plate, while E_g and E_p are the potentials of same.

Take

(3) $$\frac{C_p}{C_g} = D$$

and with good approximation

(4) $$C = C_p + C_g$$

then we get (from 1 and 2)

$$E = \frac{Q}{C} = \frac{C_g E_g + C_p E_p}{C}$$

(also with 3 and 4) $$E = \frac{C_g E_g + C_p E_p}{C_g + C_p} = \frac{E_g}{1+\frac{C_p}{C_g}} + \frac{\frac{C_p}{C_g} E_p}{1+\frac{C_p}{C_g}}$$

(5) $$E = \frac{E_g}{1+D} + \frac{D E_p}{1+D}$$

Barkhausen assigned a name to this "D", calling it "Durchgriff". The reciprocal value of this "Durchgriff" is called amplification factor in the American literature. Let us use its symbol: μ and thus

$$\frac{1}{D} = \mu \quad \text{or} \quad D = \frac{1}{\mu}$$

Thus 5. can be written

$$E = \frac{E_g + \frac{1}{\mu}E_p}{\frac{1}{\mu}+1} = \frac{\mu E_g + E_p}{1+\mu}$$

the well known expression of the effective voltage.

In most of the cases μ is large compared with unity, thus some authors - Van der Bijl, for instance - use the formula simplified

$$E \simeq \frac{\mu E_g + E_p}{\mu} = E_g + \frac{E_p}{\mu}$$

Let us follow a similar way when considering a screen grid tube

$$Q = C_g E_g + C_s E_s + C_p E_p$$

and calling $\frac{C_s}{C_g} = D_1$ $\quad \frac{C_p}{C_s} = D_2$ and $\frac{C_p}{C_g} = D_3$

and assuming that $C = C_g + C_s + C_p$, then

$$E = \frac{C_g E_g + C_s E_s + C_p E_p}{C_g + C_s + C_p} =$$

$$= \frac{E_g}{1+\frac{C_s}{C_g}+\frac{C_p}{C_g}} + \frac{\frac{C_s}{C_g}E_s}{1+\frac{C_s}{C_g}+\frac{C_p}{C_g}} + \frac{\frac{C_p}{C_s}E_p}{\frac{C_g}{C_s}+1+\frac{C_p}{C_s}} =$$

$$= \frac{E_g}{1+D_1+D_3} + \frac{D_1 E_s}{1+D_1+D_3} + \frac{D_1 D_2 E_p}{1+D_1+D_1 D_2}$$

considering that D_3 and $D_1 D_2$ are negligible in addition to unity we may write:

$$E = \frac{E_g + D_1 E_s + D_1 D_2 E_p}{1+D_1} \tag{6}$$

Even D_1 is usually small - 2-10% and thus it is possible to say that

$$E = E_g + D_1 E_s + D_1 D_2 E_p$$

the very same formula, that was used by Schottky for screen grid tubes.

We want to express 6. with the familiar terms of the amplification factor instead of those of the "Durchgriff", therefore we will take

$$D_1 = \frac{1}{\mu_1} \quad \text{and} \quad D_2 = \frac{1}{\mu_2}$$

and then (7) $$E = \frac{E_g + \frac{1}{\mu_1} E_s + \frac{1}{\mu_1 \mu_2} E_p}{1 + \frac{1}{\mu_1}} = \frac{\mu_1 E_g + E_s + \frac{E_p}{\mu_2}}{1 + \mu_1}$$

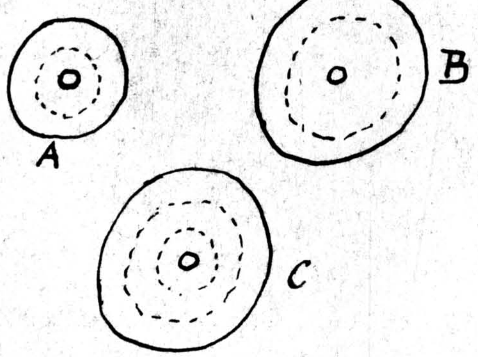
Observing this formula and comparing it with the well known formula for three element tubes, we see that the expression

$$E_s + \frac{E_p}{\mu_2}$$

took the place of the plate potential. There is a striking likeness between the following two expressions:

$$E = E_g + \frac{E_p}{\mu_1} \quad \text{and} \quad E_p' = E_s + \frac{E_p}{\mu_2}$$

The interpretation of this likeness is: (See figure).



The screen grid tube - C - might be considered as a three element tube - A - where the plate is in place of the screen grid and the plate voltage is computed as the effective voltage in - another - three element tube - B - where the screen grid took the place of the control grid*

This interpretation enables us to find a suitable formula for pentodes

tnote *The way Barkhausen arrives at the emission formula for screen grid tube is very similar to ours

Let us call

μ_1 for amplification factor between control and screen grid
μ_2 " " " " screen and suppressor grid
μ_3 " " " " suppressor grid and plate

and write 7, in the following form:

$$E_{tetrode} = \frac{\mu_1 E_g + E_s + \frac{E_p}{\mu_2}}{1 + \mu_1}$$

and then, as per analog~~r~~iam

$$E_{pentode} = \frac{\mu_1 E_g + \left[E_s + \frac{E_{sup} + \frac{E_p}{\mu_3}}{\mu_2}\right]}{1+\mu_1}$$

The effective voltage formula can be computed by simple reasoning for tubes with any number of grids, as follows:

Let us write the triode formula ~~thus~~ as:

$$E = \frac{\mu E_g + E_p}{1+\mu}$$

and consider the tetrode as a combination of two triodes:

1. Cathode - Screen grid - plate
2. Cathode - control grid - plate in place of screen grid

In the first triode:

$$E_{s,p} = \frac{\mu_2 E_s + E_p}{1+\mu_2}$$

In the second:

$$E_{g,s,p} = \frac{\mu_1 E_g + E_{s,p}}{1+\mu_1} = \frac{\mu_1 E_g + \frac{\mu_2 E_s + E_p}{1+\mu_2}}{1+\mu_1} =$$

$$= \frac{\mu_1 (1+\mu_2) E_g + \mu_2 E_s + E_p}{(1+\mu_1)(1+\mu_2)}$$

The same reasoning for pentodes:

1. Cathode - Grid #3 (suppressor) - plate
2. Cathode - Grid #2 (screen) - Plate in place of Grid #3
3. Cathode - Grid #1 (control) - plate in place of Grid #2

Then in tube 1.

$$E_{3,p} = \frac{\mu_3 E_3 + E_p}{1+\mu_3}$$

In tube 2.

$$E_{2,3,p} = \frac{\mu_2 E_2 + \frac{\mu_3 E_3 + E_p}{1+\mu_3}}{1+\mu_2}$$

In tube 3.

$$E_{1,2,3,p} = \frac{\mu_1 E_1 + \frac{\mu_2 E_2 + \frac{\mu_3 E_3 + E_p}{1+\mu_3}}{1+\mu_2}}{1+\mu_1} =$$

$$= \frac{\mu_1 (1+\mu_2)(1+\mu_3) E_1 + \mu_2 (1+\mu_3) E_2 + \mu_3 E_3 + E_p}{(1+\mu_1)(1+\mu_2)(1+\mu_3)}$$

We used the expressions "Control, Screen and Suppressor Grid" But the grids can be for any purpose, of course, for instance:

#1 - Space charge grid
#2 - Control grid
#3 - Screen grid

The same reasoning holds also for tubes with n grids (n + 2 element tubes). Then the formula:

$$E_N = \frac{\mu_1(1+\mu_2)(1+\mu_3)\dots(1+\mu_n)E_1 + \mu_2(1+\mu_3)\dots(1+\mu_n)E_2 + \dots + \mu_n E_n + E_p}{(1+\mu_1)(1+\mu_2)\dots(1+\mu_n)}$$

The tetrode is a four element tube, therefore,

$$n = 2$$

and the formula

$$E_{IV} = \frac{\mu_1(1+\mu_2)E_1 + \mu_2 E_2 + E_p}{(1+\mu_1)(1+\mu_2)}$$

This is a more precise formula than 7, because there we assumed that $\frac{1}{\mu_2}$ is very small. Taking the same assumption here, we get:

$$\frac{\mu_2}{1+\mu_2} = \frac{1}{\frac{1}{\mu_2}+1} \simeq 1 \quad \text{and} \quad 1+\mu_2 \simeq \mu_2$$

$$E_{IV} = \frac{\mu_1 E_1 + \frac{\mu_2}{1+\mu_2}E_2 + \frac{E_p}{1+\mu_2}}{1+\mu_1} = \frac{\mu_1 E_1 + E_2 + \frac{E_p}{\mu_2}}{1+\mu_1}$$

the formula which we derived previously

Having derived the effective voltage formula, the emission of the tube can be computed:

$$I_s = KE^{3/2}$$

where E is as given above

In tube designing, however, the emission, as a whole, interests us but little. It is the plate current, and its changes to which we have to pay our attention

II. PLATE CURRENT OF MULTIGRID TUBES

In a multigrid tube

$$I_s = I_1 + I_2 + \dots + I_n + I_p = KE^{3/2}$$

where

$$E = \frac{\mu_1(1+\mu_2)\dots(1+\mu_n)E_1 + \dots + \mu_n E_n + E_p}{(1+\mu_1)(1+\mu_2)\dots(1+\mu_n)}$$

In a screen grid tube N = 2

and, since the control grid (grid #1) is generally negatige against the cathode thus not drawing any current,

$$I_s = I_2 + I_p$$

The ratio of the screen grid current and plate current depends upon the dimensions of the screen grid and the plate and the voltages used. Actual tests show, however, that this ratio is independent of the control grid voltage. Let us call the ratio of the screen grid current to the plate current B. Then

$$I_s = I_2 + I_p = \left(1 + \frac{I_2}{I_p}\right) I_p = (1 + B) I_p$$

The pure scientific derivation of this B function leads to very great difficulties Therefore, we will attempt to make reasonable assumptions in order to arrive at a formula, which will enable us to figure out the ratio close enough for technical purposes

1st assumption: The current-taking capacities of grid #2 and plate are proportional to their active surface. The active surface of the grid is the cross-sectional surface, multiplied by a factor due to the space-charge around the grid wires, that of the plate is the actual surface, less the shadow effect of the grid

Let

r_2 - radius of grid #2 (center to center of wire)
r_p - radius of plate
l_2 - length of grid #2
l_p - length of plate
d_2 - diameter of grid wire
n_2 - turns per inch of grid #2
a - factor, showing the growth of the grid wire due to the space charge

We can express our first assumption:

$$B = C_1 \frac{2 r_2 \pi d_2 n_2 l_2 a}{2 r_p \pi l_p - 2 r_p \pi d_2 n_2 l_2} = C_1 \frac{a d_2 n_2 l_2 r_2}{r_p l_p - r_p d_2 n_2 l_2}$$

Let us take that plate and screen grids have the same length. This can be supposed, the more so, since the additional part of the grid has very little effect upon the performance of the tube. But then

$$B = C_1 a \frac{d_2 n_2}{1-d_2 n_2} \frac{r_2}{r_p}$$

2nd assumption: The ratio of the currents is inversely proportional with the square of the radius, which expressed:

$$B = C_2 \frac{r_p^2}{r_2^2}$$

3rd assumption: The ratio of the currents is proportional with the 3/2 power* of the ratio of the acting voltages which assumption can be expressed

$$B = C_3 \left[\frac{\mu_2 E_2 + E_p}{(1+\mu_2)E_p}\right]^{3/2}$$

All three assumptions combined will give us:

$$B = A \frac{d_2 n_2}{1-d_2 n_2} \frac{r_p}{r_2} \left[\frac{\mu_2 E_2 + E_p}{(1+\mu_2)E_p}\right]^{3/2}$$

*The 3/2 power is universal in emission problems.

A can be estimated from actual measurements, and since all other quantities are known B can be calculated

The formula, however, will not check, mainly because the secondary emission was not taken into consideration. We will not attempt to find a formula for computing the effect of the secondary emission, but will consider it as a factor, which combined with A, will be called m in our further investigations. This m is a semi-constant, which either does not change at all or but slightly with slight changes of the grid and plate construction, does not change with the control grid voltage, and changes considerably with the screen grid and plate voltages

We will see in the third part of this paper, how this m should be estimated

Thus, the formula of the plate current for the screen grid tube will take the following form

$$I_p = \frac{I_s}{B+1} = K \frac{\left[\dfrac{\mu_1(1+\mu_2)E_1 + \mu_2 E_2 + E_p}{(1+\mu_1)(1+\mu_2)}\right]^{3/2}}{m \dfrac{d_2 n_2}{1-d_2 n_2} \dfrac{r_p}{r_2} \left[\dfrac{\mu_2 E_2 + E_p}{(1+\mu_2)E_p}\right]^{3/2}}$$

Having the formula for the plate current, we can get the mutual conductance by differentiation:

$$G_m = -\frac{\partial I_p}{\partial E_1}$$

Exactly the same method can be used when computing pentode characteristics Then the plate current will be (since the suppressor current is generally 0):

$$I_p = \frac{K\left[\dfrac{\mu_1(1+\mu_2)(1+\mu_3)+\mu_2(1+\mu_3)E_2+\mu_3E_3+E_p}{(1+\mu_1)(1+\mu_2)(1+\mu_3)}\right]^{3/2}}{m\,\dfrac{d_2 n_2}{1-d_2 n_2}\,\dfrac{r_p}{r_2}\left[\dfrac{\mu_2(1+\mu_3)E_2+E_p}{(1+\mu_2)(1+\mu_3)E_p}\right]^{3/2}}$$

In order to simplify the expressions, we will introduce:

I_{IV} = plate current for screen grid tube
I_V = plate current for pentode
G_{IV} = mutual conductance for screen grid tube
G_V = mutual conductance for pentode
E_{IV} = effective voltage for screen grid tube
E_V = effective voltage for pentode
B_{IV} = ratio of screen grid current to plate current in screen grid tube
B_V = ratio of screen grid current to plate current in pentode

Then we have:

$$I_{IV} = K\frac{E_{IV}^{3/2}}{B_{IV}+1}$$

$$I_V = K\frac{E_V^{3/2}}{B_V+1}$$

$$G_{IV} = -\frac{3}{2}K\frac{E_{IV}^{1/2}}{B_{IV}+1}\,\frac{\mu_1}{1+\mu_1}$$

$$G_V = -\frac{3}{2}K\frac{E_V^{1/2}}{B_V+1}\,\frac{\mu_1}{1+\mu_1}$$

As for plate resistance and amplification factor:

$$\frac{1}{R_{IV}} = \frac{\partial I_{IV}}{\partial E_p} = \frac{3}{2}K E_{IV}^{1/2}\,\frac{\dfrac{-(B_{IV}+1)}{(1+\mu_1)(1+\mu_2)} - E_{IV}\dfrac{\partial(B_{IV}+1)}{\partial E_p}}{(B_{IV}+1)^2}$$

$$\frac{1}{R_V} = \frac{\partial I_V}{\partial E_p} = \frac{3}{2}K E_V^{1/2}\,\frac{\dfrac{-(B_V+1)}{(1+\mu_1)(1+\mu_2)(1+\mu_3)} - E_V\dfrac{\partial(B_V+1)}{\partial E_p}}{(B_V+1)^2}$$

$$\mu_{IV} = G_{IV}R_{IV} = \frac{\dfrac{\mu_1}{1+\mu_1}(B_{IV}+1)}{\dfrac{B+1}{(1+\mu_1)(1+\mu_2)} + E_{IV}\dfrac{\partial(B_{IV}+1)}{\partial E_p}}$$

$$\mu_V = G_V R_V = \frac{\dfrac{\mu_1}{1+\mu_1}(B_V+1)}{\dfrac{B+1}{(1+\mu_1)(1+\mu_2)(1+\mu_3)} + E_V\dfrac{\partial(B_{IV}+1)}{\partial E_p}}$$

Thus, we have all the formulae which are needed to calculate tetrodes and pentodes (it is a comparatively easy matter to extend the formulae for sextodes, septodes, etc). Still, their use is very limited until we have the method of finding the value of B. This method, with a few other very important faftors should be discussed in the third part of this paper

III. PRACTICAL APPLICATION OF THE FORMULAE

Though, in many cases, the above derived formulae will give results, which are very close to the readings on manufactured tubes, still, it would be entirely wrong to expect perfect results in every case. There are so many factors which were not taken into consideration

Some of them were left out in order to save very complicated computations as for instance, cooling effect of cathode connection current taking capacity of plate and grid leads, gas remainders. Some were left out because they cannot be expressed mathematically; for instance, the influence of the different coating materials, activating process, unevenness in manufacturing, etc.

To compensate for these uncontrollable variables, I use the following method:

Let us suppose, that we have a triode, the dimensions and electrical characteristics of which are known. We have to change the grid diameter, for certain reasons, and want to know what changes are required in the number of turns of the grid in order to maintain the original amplification factor. I take the simplest formula to compute the amplification factor (Van der Bijl:

$$\mu = C d r n^2 + 1$$

and substituting the known values for u, d, r and n, compute the C, which is not exactly the same for every type of tube. Then with this C I will compute the new necessary n. This method is much more accurate than taking C = 33 (the factor given by Van der Bijl for cylindrical construction) because it takes the special individual characteristic of the tube into consideration .

When computing tetrodes or pentodes, we have several such constants in our formulae. It will depend upon the problems we have to solve: Which of these constants (or semi-constants) shall be figured out. For instance, if we want or are going to change the ratio of the screen grid and plate current, the first thing shall be to find the actual value of m. In amplification factor problems the

$$\frac{\partial (B+1)}{\partial E_p}$$

constant will be important. Sometimes K has to be checked, etc

The first thing undoubtedly will generally be to determine the real control grid voltage. We know, that the voltage we apply to the control grid is modified by the contact potential. That is, a tube with -3 volts on the grid and having a contact potential of 1 volt positive, will act as a tube with 0 contact potential operated with -2 volts

In the examples we used the following method to determine the voltage which has to be added to the measured control grid voltages:

1. Let the control grid unconnected and measure the plate current

2. Connect control grid and adjust its voltage so that the plate durrent should be the same as read previously. This voltage will be the one which has to be added to the grid voltage but with opposite sign. For instance, we read -1.4 volts, the used grid voltage is -3 volts, then the real control grid voltage is -3 + 1.4 = -1.6 volts

Computing the K in the emission equation, I used the constant given by Langmuir (0.00000233). Instead of using the inside diameter of the control grid, however, I used the distance between coated surface of cathode and inside of grid, and not taken into consideration the β^2 of the original formula

Computing the partial amplification factors of the grid, Van der Bijl's formula was used, taking 33, its constant for concentrical construction

All distances are given in inches, and currents in amperes

Let us compute the plate current of a 224 tube as first example. The data is:

d1 = .006"
d2 = .006"
n1 = 26
n2 = 56
l = .625"
rc = .045"
r_1 = .075"
r_2 = .145"
rp = .406"
B = .182
E1 = -3 + 1 = -2
E2 = 90
Ep = 180

Computing the partial amplification factors:

$$\mu_1 = 33 \times .006 \times .070 \times 26^2 + 1 = 10.2$$
$$\mu_2 = 33 \times .006 \times .261 \times 56^2 + 1 = 163$$

The effective voltage:

$$E_{IV} = \frac{-10{,}2 \times 164 \times 2 + 163 \times 90 + 180}{11{,}2 \times 164} = 6{,}27$$

The plate current:

$$I_p = \frac{2{,}33 \times 2\pi \times {,}625}{{,}030 \times 10^6} \times \frac{6{,}27}{{,}182 + 1}^{3/2} = {,}00405$$

Observed plate current: $.00400$

Another example: Compute the plate current of the 57 tube:

d1 = .003"
d2 = .004"
d3 = .006"
n1 = 46
n2 = 40
n3 = 19
rc = .026"
r1 = .042"
r2 = .140"
r3 = .210"
rp = .406"
l = .625"
E1 = -3 + 1.4 = -1.6
E2 = 100
E3 = 0
Ep = 250
B = .25

The partial amplification factors:

$$\mu_1 = 33 \times .003 \times {,}098 \times 46^2 + 1 = 22$$
$$\mu_2 = 33 \times .004 \times {,}070 \times 40^2 + 1 = 16$$
$$\mu_3 = 33 \times .006 \times {,}196 \times 19^2 + 1 = 15$$

The effective voltage:

$$E_V = \frac{-22 \times 17 \times 16 \times 1{,}6 + 16 \times 16 \times 100 + 250}{23 \times 17 \times 16} = 2{,}6$$

The plate current

$$I_p = \frac{2{,}33 \times 2\pi \times {,}625}{{,}016 \times 10^6} \times \frac{2{,}6}{1 + {,}25}^{3/2} = {,}00193$$

Observed plate current: .00200

When designing a tube, we generally face problems like the following:

An experimental tube has its plate current off. What should be done in order to bring the plate current to its proper value?

In problems like this, the operating voltages are set, the length and diameters of the different grids and plate should be changed but slightly. We usually try to change the pitch of the control grid first and make other changes only if we cannot get the proper characteristic.

Let us take as our next example a power pentode. A few sample tubes were made with the following data:

n_1 = 24
k = .002513 (computed)
$\mu 1$ = 3.4 (computed)
$\mu 2$ = 9.4 (computed)
$\mu 3$ = 2.3 (computed)
E1 = -17.5 + 1 = -16.5
E2 = 162.5
E3 = 0
Ep = 162.5
I2 = .0013 (measured)
Ip = .023 (measured)
B = .0565

The effective voltage is:

$$E_v = \frac{-3.4 \times 10.4 \times 3.3 \times 16.5 + 9.4 \times 3.3 \times 162.5 + 162.5}{4.4 \times 10.4 \times 3.3} = 21.4$$

The plate current computed (to check the formula):

$$I_v = .0002513 \; \frac{21.4^{3/2}}{1.0565} = .024$$

which is very close to the measured plate current. The task was to change the turns per inch of grid #1 so that the plate current should read .017. We proceeded as follows:

K will not change, neither will B. The only change will be in the amplification factor #1. Thus we can write:

$$I_v' = .017 = \frac{.0002513}{1.0565} E_v'^{3/2}$$

$$E_v' = \left[\frac{.017 \times 1.0565}{.0002513}\right]^{2/3} = \frac{-10.4 \times 3.3 \times 16.5 \mu_1 + 9.4 \times 3.3 \times 162.5}{(1+\mu_1) 10.4 \times 3.3}$$

$$\mu_1 = 3.97$$

This is the amplification factor needed in order to get .017 plate current. But then the proper turns per inch can be computed thus:

$$3.97 = 33 \times .003 \times .041 n^2 + 1$$

$$n \simeq 27$$

Tubes were made with 27 turns per inch. The observed plate current was .017 - exactly what we aimed for.

Sometimes it will be very hard to figure out the amplification factor of a grid, as that of the control grid of a so-called variable μ tube

Our next example will be such a tube, the 58. This tube is made exactly like the 57, the data of which was given in our first example, the only difference being that the pitch of the control grid is not constant. It was desired to know the amplification factor of such a grid. Since the combined plate and screen grid current was measured as .00863 amps:

$$I_s = .00863 = .000572 \left[\frac{-17 \times 16 \times 1.6 \mu_1 + 16 \times 16 \times 100 + 250}{17 \times 16 \times (1 + \mu_1)} \right]^{3/2}$$

out of which the amplification factor:

$$\mu_1 = 11.5$$

As soon as we have this figure - all the rest being known - we can compute the mutual conductance of the tube:

$$G_m = \frac{3}{2} \frac{.000572}{1.25} \sqrt{\frac{-11.5 \times 17 \times 16 \times 1.6 + 16 \times 16 \times 100 + 250}{12.5 \times 17 \times 16}} \times \frac{11.5}{12.5} = .001560$$

while the observed Gm = .001600.

Since the mutual conductance:

$$G_m = -\frac{\partial I_p}{\partial E_g}$$

the negative value of the square-root had to be taken.

This mutual conductance checks excellently, if we consider that the mutual conductance of tubes, made in regular production will vary 30-40%. Mutual conductances, when figured with data as given in our first example, for instance, will show lower value than when measured

Let us return to the 57's

$$G_m = \frac{3}{2} K \frac{E^{1/2}}{1.25} \frac{22}{23} = .001055$$

while the published mutual conductance is .001225.

Calculating the 224's we get .000882 instead of the published 1050.

Calculating the power pentode mentioned in one of our previous examples, we get .001510 for mutual conductance, while the observed data is .001400.

Our next example will show the computation of the amplification factor of a tube. It is evident that the amplification factor of a tetrode or pentode is not independent any more from the plate voltage as it was in the case of the triodes, since the ratio of the screen and plate current changes with the plate voltage. In vain will we try to get the differential-quotient of B, since it has a variable m which is an unknown function of the plate current. Therefore, we computed the differential quotient of B, thus: measuring plate and screen current of the 57 tube at 260 and 240 plate voltage and taking their ratio, we found

$$B_{260} = .258 \quad \text{and} \quad B_{240} = .262$$

Then:

$$\frac{\Delta B}{\Delta E_p} = \frac{.262 - .258}{240 - 260} = -.0002$$

which is the differential quotient at 250 volts. This could be done, because the curvature of the B = f (Ep) curve is very slight.

Knowing this differential quotient, the plate resistance can be computed with the formula formerly given

$$\frac{1}{R_v} = \frac{3}{2} K E_v^{1/2} \frac{\frac{-(B_v+1)}{(1+\mu_1)(1+\mu_2)(1+\mu_3)} - E_v \frac{\partial(B_v+1)}{\partial E_p}}{(B_v+1)^2}$$

$$R_v = \frac{2}{3K E_v^{1/2}} \times \frac{(B_v+1)^2}{\frac{-(B_v+1)}{(1+\mu_1)(1+\mu_2)(1+\mu_3)} - E_v \frac{\partial(B_v+1)}{\partial E_p}}$$

or substituting numerical values:

$$R_p = \frac{2 \times 10^6}{3 \times 572 \times 1.63} \times \frac{1.56}{\frac{-1.25}{6256} + 2.645 \times .0002} = 3.390.000$$

and the amplification factor:

$$\mu = R_p G_m = 3.39 \times 1055 = 3575$$

The amplification factor ranges between 3000 and 4500 on manufactured tubes.

It was said, that the B is independent of the partial amplification factor of grid #1. Then its differential quotient is also independent and thus the same figure can be used on the 58 tubes:

$$R_p = \frac{2 \times 10^6}{3 \times 572 \times 2.47} \times \frac{1.56}{\frac{-1.25}{3400} + 6.1 \times .0002} = 892000$$

and the amplification factor:

$$\mu = 892000 \times ,001560 = 1392$$

Observed between 1000 and 1400.

In our last example, we will confront a rare problem. It was desired to make 58's where the ratio of plate and screen grid current should be different from the usual. The turns per inch of the screen grid was changed from 40 to 56. The formula for computing the ratio of the screen and plate current was given:

$$B = m\,\frac{dn}{1-dn}\,\frac{r_p}{r_2}\left[\frac{\mu_2(1+\mu_3)E_2+E_p}{(1+\mu_2)(1+\mu_3)E_p}\right]^{3/2}$$

Since only n was changed, the formula might be written

$$B = C\,\frac{dn}{1-dn}\left[\frac{\mu_2(1+\mu_3)E_2+E_p}{(1+\mu_2)(1+\mu_3)E_p}\right]^{3/2}$$

Calculating the partial amplification factor for 40 turns per inch

$$\mu_{40} = 33 \times ,004 \times ,070 \times 40^2 + 1 = 15,8$$

for 56 tutns per inch

$$\mu_{56} = 33 \times ,004 \times ,070 \times 56^2 + 1 = 30$$

Calculating C from B = .25 and n = 40:

$$C = \frac{,25(1-,004\times 40)}{,004\times 40}\left[\frac{16,8\times 16\times 250}{15,8\times 16\times 100+250}\right]^{3/2} = 5,6$$

Then

$$B_{56} = 5,6\,\frac{,004\times 56}{1-,004\times 56}\left[\frac{30\times 16\times 100+250}{31\times 16\times 250}\right]^{3/2} = ,393$$

Six tubes were made with these grids and B varied between .375" and .446, with an average of .397

Since the derived formulae are based on theoretical assumptions, they hold within very wide limits. Limitations have to be taken into consideration mainly on account of:

1. Deviations in manufacturing processes

2. Deviations from the constant .00000233 since this holds only for cathodes, the diameter of which is much smaller than its length and where $\frac{r_i}{r_c} > 17$ is.

3. Deviations of the actual values of the partial amplification factors from their computed values

If we are afraid, that on account of these deviations we will get deceiving results, we can eliminate them, with the method shown in the last example.

The above paper was written about ~~seven~~ ten years ago and never published because we did not succeed in proving the validity of the assumptions on which the calculation of B is based.

During the past years the writer and some of his associated (Dr. Kimball, Dr. Wright, Dr. Jervis and others) tried in vain to evolve a satisfactory formula for B. The main trouble with the one given in the paper is that since it is a semiempirical expression its differential quotient has very little meaning. Hence, the calculation of Rp and μ is not adequate

Many other people endeavored to find a formula for B (as f.i. Schottky with the "Telefunken" Company) without result

When using the formulae in connection with beam constructions, proper care should be exercised to take the unusual properties of such tubes into consideration

LUH: LRG

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ERRATA NOTICE FOR DATA FOLDER 86915

An addition has been made on the bottom of page 9 of data folder 86915 entitled, "Calculation of Receiving Tube Constants" dated September 27, 1946 by J. E. Fowler. On the last line please add a square root sign over the Vs. E_p equations to read thus:

$$ip/isg \text{ vs. } \sqrt{Ep}, \text{ and } ip/ik \text{ vs. } \sqrt{Ep}.$$

OW Pike
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J. E. Fowler
10/7/46